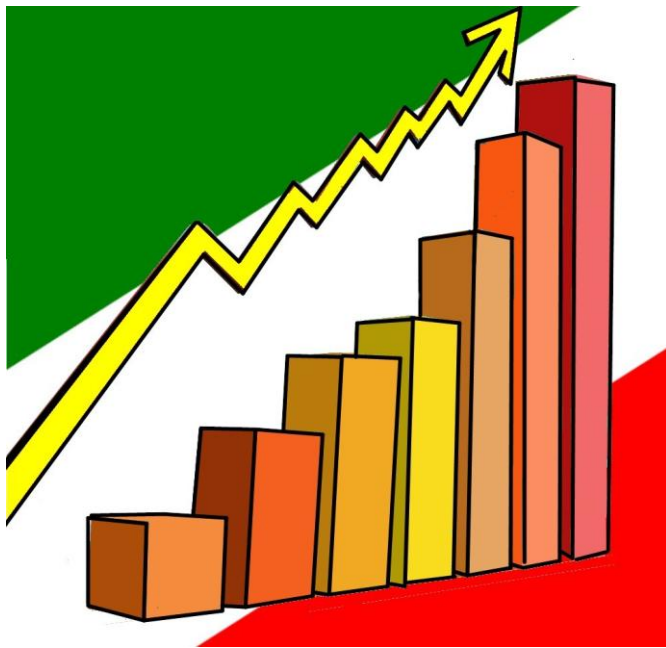


宜蘭縣 102 年度中小學科學展覽會 作品說明書

科 別：數學科

組 別：國中組



作品名稱：『增』愛找麻煩（壯圍國中）

關鍵詞：遞增數

指導老師：賴志鴻、賴麗如

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『增』愛找麻煩

摘要

經過分析，發現若一數與其平方皆為遞增數者，必具有一定的規律，較特殊的數，其分布有一定的範圍：

一、特殊的數：4 前面 n 個 3、5 前面 n 個 3、7 前面 n 個 3、7 前面 n 個 6...等，數字本身與其平方具有一定的關聯性。

二、遞增數分布範圍：

(一) n 位數中，若此數為『 $\overbrace{6666..667}^{n-1}$ 』，則此數必為此 n 位數中，所能找到符合數本身與其平方皆為遞增數中最大的。

(二) n 位數中，在『 $\overbrace{333...334}^{n-1}$ 』和『 $1\overbrace{666..667}^{n-2}$ 』之間，找不到任何符合數本身與其平方皆為遞增數的數。

(三) 末位數為 7 的數，其是遞增數的比例很大。

壹、研究動機

課堂上老師幫我們複習『根號表』的應用，在查表的過程中，發現有些數，其本身是遞增數，平方後也是遞增數，有的卻例外。很好奇怎樣的「數」才具有此特性，於是找同學一起研究，去推算本身是遞增數其平方也是遞增數的數須具備什麼條件。

貳、研究目的

算出哪些數本身為遞增數，其平方也為遞增數？這些數都分布於何處？彼此之間，又存在著哪些規律與關聯？

參、研究設備及器材

A4 紙、原子筆、電腦 (word、excel)

肆、名詞定義

遞增數：指若一三位數為 abc 其數字間 $a \leq b \leq c$ <例> 123 或 144

伍、研究過程

一、首先我們從二位數 11 開始研究

$11^2=121$	$12^2=144$	$13^2=169$	$14^2=196$	$15^2=225$
$16^2=256$	$17^2=289$	$18^2=324$	$19^2=361$	$22^2=484$
$23^2=529$	$24^2=576$	$25^2=625$	$26^2=676$	$27^2=729$
$28^2=784$	$29^2=841$	$33^2=1089$	$34^2=1156$	$35^2=1225$
$36^2=1296$	$37^2=1369$	$38^2=1444$	$39^2=1521$	$44^2=1936$
$45^2=2025$	$46^2=2116$	$47^2=2209$	$48^2=2304$	$49^2=2401$
$55^2=3025$	$56^2=3136$	$57^2=3249$	$58^2=3364$	$59^2=3481$
$66^2=4356$	$67^2=4489$	$68^2=4624$	$69^2=4761$	$77^2=5929$
$78^2=6084$	$79^2=6241$	$88^2=7744$	$89^2=7921$	$99^2=9801$

※※※發現二位數為遞增數的有 12 、 13 、 15 、 16 、 17 、 34 、 35 、 37 、 38 、 67 。

二、接著研究本身是遞增數的三位數：

$102^2=10404$	$103^2=10609$	$104^2=10816$	$105^2=11025$
$106^2=11236$	$107^2=11449$	$108^2=11664$	$109^2=11881$
$111^2=12321$	$112^2=12544$	$113^2=12769$	$114^2=12996$
$115^2=13225$	$116^2=13456$	$117^2=13689$	$118^2=13924$
$119^2=14161$	$122^2=14884$	$123^2=15129$	$124^2=15376$
$125^2=15625$	$126^2=15876$	$127^2=16129$	$128^2=16384$
$129^2=16641$	$133^2=17689$	$134^2=17956$	$135^2=18225$
$136^2=18496$	$137^2=18769$	$138^2=19044$	$139^2=19321$
$144^2=20736$	$145^2=21025$	$146^2=21316$	$147^2=21609$
$148^2=21904$	$149^2=22201$	$155^2=24025$	$156^2=24336$
$157^2=24649$	$158^2=24964$	$159^2=25281$	$166^2=27556$
$167^2=27889$	$168^2=28224$	$169^2=28561$	$177^2=31329$
$178^2=31684$	$179^2=32041$	$188^2=35344$	$189^2=35721$

$199^2=39601$	$222^2=49284$	$223^2=49729$	$224^2=50176$
$225^2=50625$	$226^2=51076$	$227^2=51529$	$228^2=51984$
$229^2=52441$	$233^2=54289$	$234^2=54756$	$235^2=55225$
$236^2=55696$	$237^2=56169$	$238^2=56644$	$239^2=57121$
$244^2=59536$	$245^2=60025$	$246^2=60516$	$247^2=61009$
$248^2=61504$	$249^2=62001$	$255^2=65025$	$256^2=65536$
$257^2=66049$	$258^2=66564$	$259^2=67081$	$266^2=68644$
$267^2=71289$	$268^2=71824$	$269^2=72361$	$277^2=76729$
$278^2=77284$	$279^2=67081$	$288^2=82944$	$289^2=83521$
$299^2=89401$	$333^2=110889$	$334^2=111556$	$335^2=112225$
$336^2=112896$	$337^2=113569$	$338^2=114244$	$339^2=114921$
$344^2=118336$	$345^2=119025$	$346^2=119716$	$347^2=120409$
$348^2=121104$	$349^2=121801$	$355^2=126025$	$356^2=126736$
$357^2=127449$	$358^2=128164$	$359^2=128881$	$366^2=133956$
$367^2=134689$	$368^2=135424$	$369^2=136161$	$377^2=142129$
$378^2=142884$	$379^2=143641$	$388^2=150544$	$389^2=151321$
$399^2=159201$	$444^2=197136$	$445^2=198025$	$446^2=198916$
$447^2=199809$	$448^2=200704$	$449^2=201601$	$455^2=207025$
$456^2=207936$	$457^2=208849$	$458^2=209764$	$459^2=210681$
$466^2=217156$	$467^2=218089$	$468^2=219024$	$469^2=219961$
$477^2=227529$	$478^2=228484$	$479^2=229441$	$488^2=238144$
$489^2=239121$	$499^2=249001$	$555^2=308025$	$556^2=309136$
$557^2=310249$	$558^2=311364$	$559^2=312481$	$566^2=320356$
$567^2=321489$	$568^2=322624$	$569^2=323761$	$577^2=332929$
$578^2=334084$	$579^2=335241$	$588^2=345744$	$589^2=346921$
$599^2=358801$	$666^2=443556$	$667^2=444889$	$668^2=446224$
$669^2=44756$	$1677^2=458329$	$678^2=459684$	$679^2=461041$
$688^2=473344$	$689^2=474721$	$699^2=488601$	$777^2=603729$

$778^2=605284$	$779^2=606841$	$788^2=620944$	$789^2=622521$
$799^2=638401$	$888^2=788544$	$889^2=790321$	$899^2=808201$
$999^2=998001$			

※※※發現三位數為遞增數的有 116、117、167、334、335、337、367、667。

三、有了三位數的結果，於是我們繼續研究四位數：

$1111^2=1234321$	$1112^2=1236544$	$1113^2=1238769$	$1114^2=1240996$
$1115^2=1243225$	$1116^2=1245456$	$1117^2=1247689$	$1118^2=1249924$
$1119^2=1252161$	$1122^2=1258884$	$1123^2=1261129$	$1124^2=1263376$
$1125^2=1265625$	$1126^2=1267876$	$1127^2=1270129$	$1128^2=1272384$
$1129^2=1274641$	$1133^2=1283689$	$1134^2=1285956$	$1135^2=1288225$
$1136^2=1290496$	$1137^2=1292769$	$1138^2=1295044$	$1139^2=1297321$
$1144^2=1308736$	$1145^2=1311025$	$1146^2=1313316$	$1147^2=1315609$
$1148^2=1317904$	$1149^2=1320201$	$1155^2=1334025$	$1156^2=1336336$
$1157^2=1338649$	$1158^2=1340964$	$1159^2=1343281$	$1166^2=1359556$
$1167^2=1361889$	$1168^2=1364224$	$1169^2=136656$	$1177^2=1385329$
$1178^2=1387684$	$1179^2=1390041$	$1188^2=1411344$	$1189^2=1413721$
$1199^2=1437601$	$1222^2=1493284$	$1223^2=1495729$	$1224^2=1498176$
$1225^2=1500625$	$1226^2=1503076$	$1227^2=1505529$	$1228^2=1507984$
$1229^2=1510441$	$1233^2=1520289$	$1234^2=1522756$	$1235^2=1525225$
$1236^2=1527696$	$1237^2=1530169$	$1238^2=1532644$	$1239^2=1535121$
$1244^2=1547536$	$1245^2=1550025$	$1246^2=1552516$	$1247^2=1555009$
$1248^2=1557504$	$1249^2=1560001$	$1255^2=1575025$	$1256^2=1577536$
$1257^2=1580049$	$1258^2=1582564$	$1259^2=1585081$	$1266^2=1602756$
$1267^2=1605289$	$1268^2=1607824$	$1269^2=1610361$	$1277^2=1630729$
$1278^2=1633284$	$1279^2=1635841$	$1288^2=1658944$	$1289^2=1661521$
$1299^2=1687401$	$1333^2=1776889$	$1334^2=1779556$	$1335^2=1782225$

1336 ² =1784896	1337 ² =1787569	1338 ² =1790244	1339 ² =1792921
1344 ² =1806336	1345 ² =1809025	1346 ² =1811716	1347 ² =1814409
1348 ² =1817104	1349 ² =1819801	1355 ² =1836025	1356 ² =1838736
1357 ² =1841449	1358 ² =1844164	1359 ² =1846881	1366 ² =1865956
1367 ² =1868689	1368 ² =1871424	1369 ² =1874161	1377 ² =1896129
1378 ² =1898884	1379 ² =1901641	1388 ² =1926544	1389 ² =1929321
1399 ² =1957201	1444 ² =2085136	1445 ² =2088025	1446 ² =2090916
1447 ² =2093809	1448 ² =2096704	1449 ² =2099601	1455 ² =2117025
1456 ² =2119936	1457 ² =2122849	1458 ² =2125764	1459 ² =2128681
1466 ² =2149156	1467 ² =2152089	1468 ² =2152089	1469 ² =2157961
1477 ² =2181529	1478 ² =2184484	1479 ² =2187441	1488 ² =2214144
1489 ² =2217121	1499 ² =2220100	1555 ² =2418025	1556 ² =2421136
1557 ² =2424249	1558 ² =2427364	1559 ² =2430481	1566 ² =2452356
1567 ² =2455489	1568 ² =2458624	1569 ² =2461761	1577 ² =2486929
1578 ² =2490084	1579 ² =2493241	1588 ² =2521744	1589 ² =2524921
1599 ² =2556801	1666 ² =2775556	1667²=2778889	1668 ² =2782224
1669 ² =2785561	1677 ² =2812329	1678 ² =2815684	1679 ² =2819041
1688 ² =2849344	1689 ² =2852721	1699 ² =2886601	1777 ² =3157729
1778 ² =3161284	1779 ² =3164841	1788 ² =3196944	1789 ² =3200521
1799 ² =3236401	1888 ² =3564544	1889 ² =3568321	1899 ² =3606201
1999 ² =3996001	2222 ² =4937284	2223 ² =4941729	2224 ² =4946176
2225 ² =4950625	2226 ² =4955076	2227 ² =4959529	2228 ² =4963984
2229 ² =4968441	2233 ² =4986289	2234 ² =4990756	2235 ² =4995225
2236 ² =4999696	2237 ² =5004169	2238 ² =5008644	2239 ² =5013121
2244 ² =5035536	2245 ² =5040025	2246 ² =5044516	2247 ² =5049009
2248 ² =5053504	2249 ² =5058001	2255 ² =5085025	2256 ² =5089536
2257 ² =5094049	2258 ² =5098564	2259 ² =5103081	2266 ² =5134756
2267 ² =5139289	2268 ² =5143824	2269 ² =5148361	2277 ² =5184729

2278 ² =5189284	2279 ² =5193841	2288 ² =5234944	2289 ² =5239521
2299 ² =5285401	2333 ² =5442889	2334 ² =5447556	2335 ² =5452225
2336 ² =5456896	2337 ² =5461569	2338 ² =5466244	2339 ² =5470921
2344 ² =5494336	2345 ² =5499025	2346 ² =5503716	2347 ² =5508409
2348 ² =5513104	2349 ² =5517801	2350 ² =5522500	2355 ² =5546025
2356 ² =5550736	2357 ² =5555449	2358 ² =5560164	2359 ² =5564881
2366 ² =5597956	2367 ² =5602689	2368 ² =5607424	2369 ² =5612161
2377 ² =5650129	2378 ² =5654884	2379 ² =5659641	2388 ² =5702544
2389 ² =5707321	2399 ² =5755201	2444 ² =5973136	2445 ² =5978025
2446 ² =5982916	2447 ² =5987809	2448 ² =5992704	2449 ² =5997601

⋮
⋮

3333 ² =11108889	3334²=11115556	3335²=11122225	3336 ² =11128896
3337²=11135569	3338 ² =11142244	3339 ² =11148921	3344 ² =11182336
3345 ² =11189025	3346 ² =11195716	3347 ² =11202409	3348 ² =11209104
3349 ² =11215801	3355 ² =11256025	3356 ² =11262736	3357 ² =11269449
3358 ² =11276164	3359 ² =11282881	3366 ² =11329956	3367²=11336689
3368 ² =11343424	3369 ² =11350161	3376 ² =11397376	3377 ² =11404129
3378 ² =11410884	3379 ² =11417641	3388 ² =11478544	3389 ² =11485321
3399 ² =11553201	3444 ² =11861136	3445 ² =11868025	3446 ² =11874916

⋮
⋮

3666 ² =13439556	3667²=13446889	3668 ² =13454224	3669 ² =13461561
3677 ² =13520329	3678 ² =13527684	3679 ² =13535041	3688 ² =13601344
3689 ² =13608721	3699 ² =13682601	3777 ² =14265729	3778 ² =14273284
3779 ² =14280841	3788 ² =14348944	3789 ² =14356521	3799 ² =14432401
3888 ² =15116544	3889 ² =15124321	3899 ² =15202201	3999 ² =15992001
4444 ² =19749136	4445 ² =19758025	4446 ² =19766916	4447 ² =19775809

$4448^2=19784704$	$4449^2=19793601$	$4455^2=19847025$	$4456^2=19855936$
$4457^2=19864849$	$4458^2=19873764$	$4459^2=19882681$	$4466^2=19945156$
$4467^2=19954089$	$4468^2=19963024$	$4469^2=19971961$	$4477^2=20043529$
$4478^2=20052484$	$4479^2=20061441$	$4488^2=20142144$	$4489^2=20151121$
$4499^2=20241001$	$4555^2=20748025$	$4556^2=20757136$	$4557^2=20766249$
	:		
	:		
$6666^2=44435556$	$6667^2=44448889$	$6668^2=44462224$	$6669^2=44475561$
$6677^2=44582329$	$6678^2=44595684$	$6679^2=44609041$	$6688^2=44729344$
$6689^2=44742721$	$6699^2=44876601$	$6777^2=45927729$	$6778^2=45941284$
$6789^2=46090521$	$6799^2=46226401$	$6888^2=47444544$	$6889^2=47458321$
$6899^2=47596201$	$6999^2=48986001$	$7777^2=60481729$	$7778^2=60497284$
$7779^2=60512841$	$7788^2=60652944$	$7789^2=60668521$	$7799^2=60824401$
$7888^2=62220544$	$7889^2=62236321$	$7999^2=63984001$	$8888^2=78996544$
$8889^2=79014321$	$8899^2=79192201$	$8999^2=80982001$	$9999^2=99980001$

※※※發現四位數為遞增數的有 **1667**、**3334**、**3335**、**3337**、**3367**、**3667**、**6667**。

五、接著我們繼續討論五位數

$11111^2=123454321$	$11112^2=123476544$	$11113^2=123498769$
$11114^2=123520996$	$11115^2=123543225$	$11116^2=123565456$
$11117^2=123587689$	$11118^2=123609924$	$11119^2=123632161$
$11122^2=123698884$	$11123^2=123721129$	$11124^2=123743376$
$11125^2=123765625$	$11126^2=123787876$	$11127^2=123810129$
$11128^2=123832384$	$11129^2=123854641$	$11133^2=123943689$
$11134^2=123965956$	$11135^2=123988225$	$11136^2=124010496$
$11137^2=124032769$	$11138^2=124055044$	$11139^2=124077321$
$11144^2=124188736$	$11145^2=124211025$	$11146^2=124233316$
$11147^2=124255609$	$11148^2=124277904$	$11149^2=124300201$

$11155^2=124434025$	$11156^2=124456336$	$11157^2=124478649$
$11158^2=124500964$	$11159^2=124523281$	$11166^2=124679556$
$11167^2=124701889$	$11168^2=124724224$	$11169^2=124746561$
$11177^2=124925329$	$11178^2=124947684$	$11179^2=124970041$
$11188^2=125171344$	$11189^2=125193721$	$11199^2=125417601$
$11222^2=125933284$	$11223^2=125955729$	$11224^2=125978176$
$11225^2=126000625$	$11226^2=126023076$	$11227^2=126045529$
$11228^2=126067984$	$11229^2=126090441$	$11233^2=126180289$
$11234^2=126202756$	$11235^2=126225225$	$11236^2=126247696$
$11237^2=126270169$	$11238^2=126292644$	$11239^2=126315121$
$11244^2=126427536$	$11245^2=126450025$	$11246^2=126472516$
$11247^2=126495009$	$11248^2=126517504$	$11249^2=126540001$
$11255^2=126675025$	$11256^2=126697536$	$11257^2=126720049$
$11258^2=126742564$	$11259^2=126765081$	$11266^2=126922756$
$11267^2=126945289$	$11268^2=126967824$	$11269^2=126990361$
$11277^2=127170729$	$11278^2=127193284$	$11279^2=127215841$
$11288^2=127418944$	$11289^2=127441521$	$11299^2=127667401$
$11333^2=128436889$	$11334^2=128459556$	$11335^2=128482225$
$11336^2=128504896$	$11337^2=128527569$	$11338^2=128550244$
$11339^2=128572921$	$11344^2=128686336$	$11345^2=128709025$
$11346^2=128731716$	$11347^2=128754409$	$11348^2=128777104$
$11349^2=128799801$	$11355^2=128936025$	$11356^2=128958736$
$11357^2=128981449$	$11358^2=129004164$	$11359^2=129026881$
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※※※發現五位數為遞增數的有 16667、33334、33335、33337、33367、33667、36667、66667。

六、接著我們繼續觀察六位數

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$667889^2=446075716321$	$667999^2=446222664001$	$668888^2=447411156544$
$668889^2=447412494321$	$669999^2=448898660001$	$677777^2=459381661729$
$677778^2=459383017284$	$677779^2=459384372841$	$677789^2=459397928521$
$677799^2=459411484401$	$677888^2=459532140544$	$677889^2=459533496321$
$677999^2=459682644001$	$678888^2=460888916544$	$678889^2=460890274321$
$678899^2=460903852201$	$678999^2=461039642001$	$679999^2=462398640001$
$688888^2=474566676544$	$688889^2=474568054321$	$688899^2=474581832201$
$688999^2=474719622001$	$689999^2=476098620001$	$699999^2=489998600001$
$677999^2=459682644001$	$678888^2=460888916544$	$678889^2=460890274321$
$678899^2=460903852201$	$678999^2=461039642001$	$679999^2=462398640001$
$777777^2=604937061729$	$777778^2=604938617284$	$777779^2=604940172841$
$777788^2=604954172944$	$777789^2=604955728529$	$778899^2=606683652201$
$778999^2=606839442001$	$779999^2=608398440001$	$788888^2=622344276544$
$788889^2=622345854321$	$788899^2=622361632201$	$788999^2=622519422001$
$789999^2=624098420001$	$799999^2=639998400001$	$888888^2=790121876544$
$888889^2=790123654321$	$888899^2=790141432201$	$888999^2=790319222001$
$889999^2=792098220001$	$899999^2=809998200001$	$999999^2=999998000001$

※※※發現六位數為遞增數的有 **16667**、**33334**、**33335**、**33337**、**33367**、**33367**、**33667**、**36667**、**66667**。

陸、研究結果

探討完六位數後，我們發現很多有趣的現象：

一、對某些特定數字而言：

(一)4 前面 n 個 3：

$$34^2=1156$$

$$334^2=111556$$

$$3334^2=11115556$$

⋮

※※其平方後的值為

$(n+1)$ 個 1	n 個 5	6
-------------	-------	---

未位數

<例> $\overbrace{3333333333}^{10} 34^2 = \overbrace{1111111111}^{11} \overbrace{5555555555}^{10} 6$

(二)5 前面 n 個 3 :

$$35^2=1225$$

$$335^2=112225$$

$$3335^2=11122225$$

⋮

※※其平方後的值為

n 個 1	$(n+1)$ 個 2	5
-------	-------------	---

未位數

<例> $\overbrace{3333333333}^{10} 35^2 = \overbrace{1111111111}^{10} \overbrace{2222222222}^{11} 5$

(三)7 前面 n 個 3 :

$$37^2=1369$$

$$337^2=113569$$

$$3337^2=11135569$$

⋮

※※其平方後的值為

n 個 1	3	$(n-1)$ 個 5	69
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未二位數

<例> $\overbrace{3333333333}^{10} 37^2 = \overbrace{1111111111}^{10} 3 \overbrace{5555555555}^9 69$

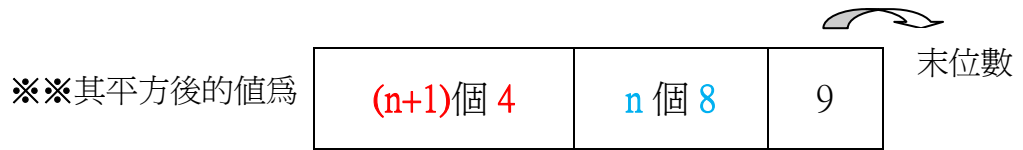
(四)7 前面 n 個 6 :

$$67^2=4489$$

$$667^2=444889$$

$$6667^2=44448889$$

⋮



<例> $\overbrace{6666666666}^{10} 67^2 = \overbrace{4444444444}^{11} \overbrace{8888888888}^{10} 9$

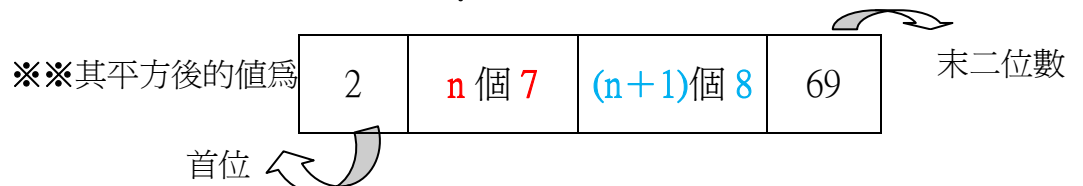
(五)1 與 7 之間 n 個 6 :

$$167^2=27889$$

$$1667^2=2778889$$

$$16667^2=277788889$$

⋮



<例> $1\overbrace{6666666666}^{10} 67^2 = 2\overbrace{7777777777}^{10} \overbrace{8888888888}^{11} 9$

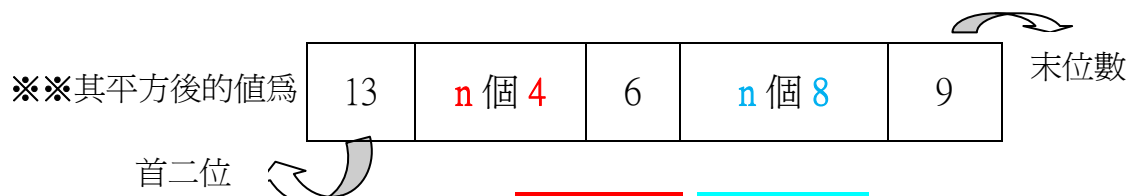
(六)3 與 7 之間 n 個 6 :

$$367^2=134689$$

$$3667^2=13446889$$

$$36667^2=1344468889$$

⋮



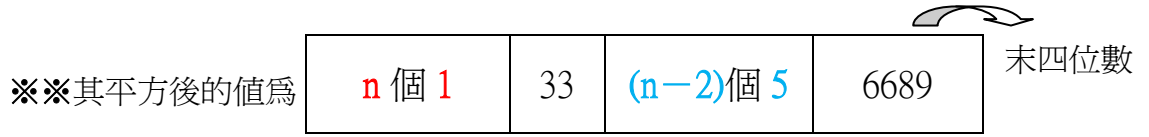
<例> $3\overbrace{6666666666}^{10} 67^2 = 13\overbrace{4444444444}^{10} 6\overbrace{8888888888}^{10} 9$

(七) 67 前面 $n \geq 2$ 個 3 : (即除 367 外)

$$3367^2=11336689$$

$$33367^2=1113356689$$

$$333367^2=111133556689$$



<例> $\overbrace{3333333333}^{10} 367^2 = \overbrace{1111111111}^{10} 33 \overbrace{55555555}^8 6689$

二、遞增數分布範圍：

從二位數可發現 17 到 34 之間沒有任何一個遞增數，從三位數同樣可發現這個現象，167 至 334 間沒有任何一個遞增數，從四位數亦可發現，1667 至 3334 間沒有任何一個遞增數，在 n 位數中，在 $\lceil \overbrace{333\dots334}^{n-1} \rceil$ 和 $\lceil \overbrace{1666\dots667}^{n-2} \rceil$ 之間，找不到任何符合數本身與其平方皆為遞增數的數。

三、從觀察找出的遞增數可發現：

在 n 位數中，若此數為 $\lceil \overbrace{6666\dots667}^{n-1} \rceil$ ，則此數必為此 n 位數中，所能找到符合數本身與其平方皆為遞增數中最大的。

四、 n 位數的遞增數數量關係：

二位數共 10 個、三位數共 8 個、四位數共 7 個、五位數共 8 個、六位數共 9 個，二位至四位的遞增數數量逐漸減少，四位數後逐漸增加，而且有越來越多的趨勢。

柒、討論

一、遞增數的出現是否有著一定規律？

結果：一開始全靠土法煉鋼，看不出規律，四位數前所找到的遞增數，完全是把數字全部印出，從表格一行一列找出的，四位數後才逐漸看出規律，為了確認規律我們持續做到六位數，果然發現遞增數中，第四位前有些不具規律，但從第四位起，所有遞增數皆有規律，沒有例外。

二、是否能以推算的方式找出下一個遞增數，而不用從全部數字當中去找？

結果：找出遞增數出現的規律後，即可推算出下一個遞增數為何，且可以排除幾個範圍內的數字，增快尋找遞增數時的速度。

捌、結論

- 一、藉由本文探討，可知遞增數出現的範圍及彼此之間的規律，以及遞增數與其平方的關聯。
- 二、我們只能利用觀察比較的方法去找到遞增數的規律性，至於如何去驗證遞增數，是我們未來追求的目標，國中所學的尚不足以證明，只能待往後學習到時，再進行證明。
- 三、從尋找遞增與其平方的規律的過程中，學習如何應用所學的數學概念進行思考、推測、探討、歸納，並在過程中領會學習數學的樂趣，從瓶頸跳脫的成就。

玖、參展心得及活動花絮

剛開始在研究時，行動、想法都非常踴躍的要嘗試，想尋找任何的可能性，可是不久就遇上了瓶頸，望著看似毫無規律的數字，完全摸不著頭緒，當時愈急著找出規律頭腦就愈混亂，找到後來，一度想放棄這個題材，可又想已費了不少力氣去做這題目，這樣放棄實在可惜。於是在老師們的鼓勵指點下，不死心的繼續堅持研究，所幸付出的努力沒有白費，我們發現遞增數其實具備許多規律，在找到規律後，心中還蠻有成就感的，但較為可惜的是，因我們在國中階段所學的，無法進行更深入的證明研究，希望將來有能力做進一步的深入探討。



輸入資料



老師協助指導



將遞增數找出來

